

Ride Share Allocation



Student Worksheet - Answers

7 8 9 10 11 12



TI-Nspire



Worksheet



Student



30 min

Introduction

Zippy-Roo Ride Share company has five riders that have just registered for rides: Angel, Bella, Chris, Dion and Eric. Wait times per vehicle ($C_1, C_2 \dots C_5$) to reach each rider are as follows:

- Angel = {4, 5, 7, 6, 9}
- Bella = {5, 4, 3, 7, 11}
- Chris = {3, 6, 5, 5, 7}
- Dion = {8, 7, 6, 5, 6}
- Eric = {9, 3, 4, 5, 5}



Zippy-Roo do not have a car pool option, so only one rider can be allocated to each vehicle. Zippy-Roo's goal is to minimise the total amount of wait time.

Question: 1.

Suppose the following allocations are made: Angel = Car 1 (C_1), Bella = Car 2, Chris = Car 3, Dion = Car 4 and Eric = Car 5. What would be the total amount of wait time?

Answer: $4 + 4 + 5 + 5 + 5 = 23$ minutes

Question: 2.

Make up your own allocation (guess) to see if you can improve on the result from Question 1.

Answer: Answers will vary dependent on selections. Possible answer: $4(C_1) + 3(C_3) + 5(C_4) + 6(C_6) + 3(C_2) = 21$

Hungarian Algorithm

The Hungarian Algorithm is a tool which can be used to solve these types of problems. Allocating five vehicles could be done with trial and error, however, as the number of vehicles increases, so too does the number of possibilities.

If you have not used the Hungarian Algorithm before, scan the QR code opposite to watch a video to help:



- Understand how and what the algorithm does
- Apply the algorithm to solve an allocation problem
- How to solve it on your calculator.

Question: 3.

Express the problem information in a matrix.

Answer: $m = \begin{bmatrix} 4 & 5 & 7 & 6 & 9 \\ 5 & 4 & 3 & 7 & 11 \\ 3 & 6 & 5 & 5 & 7 \\ 8 & 7 & 6 & 5 & 6 \\ 9 & 3 & 4 & 5 & 5 \end{bmatrix}$

Question: 4.

Complete the row reduction (Step 1) of the algorithm.

$$\text{Answer: } m = \begin{bmatrix} 0 & 1 & 3 & 2 & 5 \\ 2 & 1 & 0 & 4 & 8 \\ 0 & 3 & 2 & 2 & 4 \\ 3 & 2 & 1 & 0 & 1 \\ 6 & 0 & 1 & 2 & 2 \end{bmatrix} \begin{matrix} 4 \\ 3 \\ 3 \\ 5 \\ 3 \end{matrix}$$

Question: 5.

Complete the column reduction (Step 2) of the algorithm.

$$\text{Answer: } m = \begin{bmatrix} 0 & 1 & 3 & 2 & 4 \\ 2 & 1 & 0 & 4 & 7 \\ 0 & 3 & 2 & 2 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 6 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Question: 6.

There is more than one way to complete Step 3 (refer video), choose one and proceed.

Note: This step may have to be repeated, record each iteration until allocation is possible.

Answers: (options for first iteration) – Mark off Columns or Rows with multiple zeros first, then ...

Process rows:

$$m = \begin{bmatrix} 0 & 1 & 3 & 2 & 4 \\ 2 & 1 & 0 & 4 & 7 \\ 0 & 3 & 2 & 2 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 6 & 0 & 1 & 2 & 1 \end{bmatrix}$$

Process columns:

$$m = \begin{bmatrix} 0 & 1 & 3 & 2 & 4 \\ 2 & 1 & 0 & 4 & 7 \\ 0 & 3 & 2 & 2 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 6 & 0 & 1 & 2 & 1 \end{bmatrix}$$

Random

$$m = \begin{bmatrix} 0 & 1 & 3 & 2 & 4 \\ 2 & 1 & 0 & 4 & 7 \\ 0 & 3 & 2 & 2 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 6 & 0 & 1 & 2 & 1 \end{bmatrix}$$

Permutations from here continue to increase with each successive iteration, however, the algorithm will still optimise the total wait time once completed.

Question: 7.

Complete the allocation.

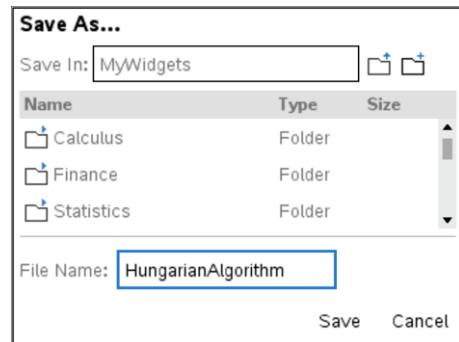
Note: More than one allocation is possible. Show at least two solutions to illustrate that the overall wait time is the same for both.

$$\text{Answer: } m = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{Note: This is only one solution/allocation. Total Wait time: 21 minutes}$$

Calculator Instructions

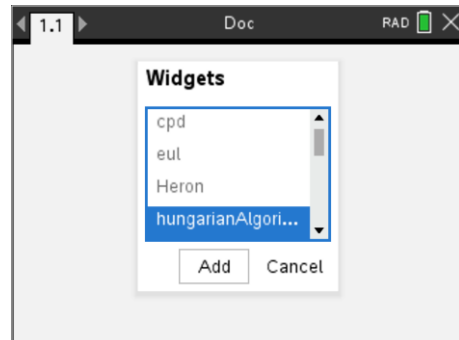
Open the HungarianAlgorithm file.

Save the file to your MyWidgets folder.



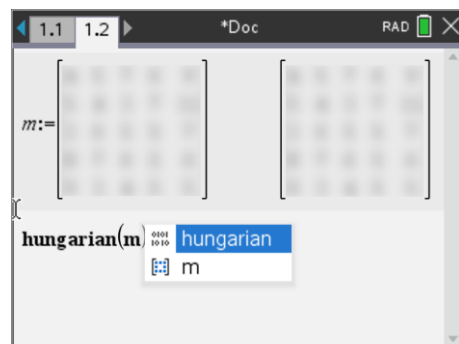
Start a new document and select **"Widget"** from the insert menu.

Scroll down until you find the Hungarian Algorithm Widget and select **Add**.



Follow the instructions (Insert Calculator Application), define the matrix using the rider – vehicle wait times and then run the Hungarian Algorithm Program from the Variables menu.

Note: Values in the matrices have been blurred intentionally.



When entering values in the matrix, use the **tab** to navigate.

Question: 8.

How does the calculator solution compare with the one that you completed by hand? [Total wait time and allocation]

Answers: The total time 21 minutes is the same. Allocations will vary depending on priorities.

Calculator Solution:

Angel = C₂, Bella C₃, Chris = C₁,
Dion = C₄ Eric = C₅ {5, 3, 3, 5, 5} = 21 minutes.

Alternate solution:

Angel = C₁, Bella C₃, Chris = C₄,
Dion = C₅ Eric = C₂ {3, 4, 5, 6, 3} = 21 minutes.

